**Gettysburg Address**

(This activity is adapted from the Sampling Words activity by Beth Chance and Allan Rossman.)

Suppose we wish to know something about a population. For example, we might want to know the average height of a 12 year-old boy, the proportion of Americans over 70 who send text messages, or the typical number of kittens in a litter. It is often not possible or practical to collect data from the entire **population**, so instead, we collect data from a **sample** of the population. If our sample is **representative** of the population, we can make **inferences**, or in other words, draw conclusions about the population.

The Gettysburg Address is one of the most famous speeches in American history. On average, how long were the words that President Lincoln used in his speech?

Rather than count the number of letters in every word of the Gettysburg Address so that we can find an average, in this activity we will try to learn about this population of 268 words by taking samples of words from the speech.

Gettysburg Address

by Abraham Lincoln

November 19, 1863

|  |
| --- |
| Four score and seven years ago, our fathers brought forth upon this continent a new nation: conceived in liberty, and dedicated to the proposition that all men are created equal.  Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battlefield of that war.  We have come to dedicate a portion of that field as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.  But, in a larger sense, we cannot dedicate, we cannot consecrate, we cannot hallow this ground. The brave men, living and dead, who struggled here have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember, what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us, that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion, that we here highly resolve that these dead shall not have died in vain, that this nation, under God, shall have a new birth of freedom, and that government of the people, by the people, for the people, shall not perish from the earth. |

Part I

1. Circle 10 words from the Gettysburg Address that you think provide a representative sample. Record your ten words in the table below and count the number of letters in each word.

|  |  |  |
| --- | --- | --- |
|  | **Word** | **Number**  **of Letters** |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |

1. Do you think the ten words in your sample are representative of the 268 words in the population? Explain briefly.
2. Create a dot plot of your sample.
3. What are the observational units? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. Calculate the mean for your sample.

mean = \_\_\_\_\_\_\_\_\_\_\_\_\_

1. Is this mean a parameter or a statistic? \_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Record your mean on the table provided on the chalkboard and then place a dot for your sample mean on the class dot plot.
3. Find the mean of all the samples for all your classmates.

mean of all the class samples = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Ask your teacher what the actual mean word length is for the 268 words in the Gettysburg Address.

mean word length for population: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Is this a parameter or a statistic? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Is the mean of all the class samples close to the population mean? \_\_\_\_\_\_\_
2. How many of the students in your class had samples that were greater than the actual population mean?\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What proportion of the class is this? \_\_\_\_\_\_\_\_\_\_\_\_

How do we ensure that we select a sample that is representative of the population? We choose a method that eliminates the possibility that our own preferences, favoritism or **biases** impact who (or what) is selected. We want to give all individuals (or objects) whether old or young, black or white, small or large an equal chance to be chosen. We do not want the method of picking the sample to produce samples with characteristics that are different than the population characteristics.

1. In what way were our samples different from the population?

A sampling method that helps us to avoid biases is to select a **simple random sample**. If we want a sample to have *n* individuals, we use a method that will ensure that every possible sample from the population of size *n* has an equal chance of being selected.

Part II

1. On page 6 of this activity, you will see a numbered list of all of the words from the Gettysburg Address. Your teacher will explain how to use a random digits table or a random number generator to select a random sample of 10 words. First, select 10 random numbers from 1 to 268. Record the numbers in the table below in the *ID Number* column. Identify the word that corresponds to each ID number and record the word and its length in the table.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **ID Number** | **Word** | **Number**  **of Letters** |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |

1. Create a dot plot of your sample.
2. Calculate the mean for your sample.

mean = \_\_\_\_\_\_\_\_\_\_\_\_\_

1. Which is closer to the actual population mean: the mean of your judgment sample or the mean of your random sample?
2. Record your mean on the table provided on the chalkboard and then place a dot for your sample mean on the class dot plot.
3. Find the mean of all the samples for all your classmates.

mean of all the class samples = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Is the mean of all the random samples close to the population mean? \_\_\_\_\_\_\_
2. How does the distribution of sample averages from the random samples compare to that from your “circle ten words” sample distribution?
3. Do the sample averages from the random samples tend to over- or under-estimate the population average, or are they roughly split evenly on both sides?

Conclusion

1. What was the population in this activity? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. What characteristic (parameter) of the population were we trying to learn about in this activity?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. What were our samples for this activity? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. What sample statistic did we calculate in this activity? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Which sampling method provided the best estimate of the mean word size for the population of all words in the Gettysburg Address?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **ID** | **Word** | **ID** | **Word** | **ID** | **Word** | **ID** | **Word** | **ID** | **Word** | **ID** | **Word** | **ID** | **Word** | **ID** | **Word** |  |
| 1 | Four | 35 | in | 69 | dedicate | 103 | But | 137 | add | 171 | here | 205 | these | 239 | that |  |
| 2 | score | 36 | a | 70 | a | 104 | in | 138 | or | 172 | to | 206 | honored | 240 | this |  |
| 3 | and | 37 | great | 71 | portion | 105 | a | 139 | detract | 173 | the | 207 | dead | 241 | nation |  |
| 4 | seven | 38 | civil | 72 | of | 106 | larger | 140 | The | 174 | unfinished | 208 | we | 242 | under |  |
| 5 | years | 39 | war | 73 | that | 107 | sense | 141 | world | 175 | work | 209 | take | 243 | God |  |
| 6 | ago | 40 | testing | 74 | field | 108 | we | 142 | will | 176 | which | 210 | increased | 244 | shall |  |
| 7 | our | 41 | whether | 75 | as | 109 | cannot | 143 | little | 177 | they | 211 | devotion | 245 | have |  |
| 8 | fathers | 42 | that | 76 | a | 110 | dedicate | 144 | note | 178 | who | 212 | to | 246 | a |  |
| 9 | brought | 43 | nation | 77 | final | 111 | we | 145 | nor | 179 | fought | 213 | that | 247 | new |  |
| 10 | forth | 44 | or | 78 | resting | 112 | cannot | 146 | long | 180 | here | 214 | cause | 248 | birth |  |
| 11 | upon | 45 | any | 79 | place | 113 | consecrate | 147 | remember | 181 | have | 215 | for | 249 | of |  |
| 12 | this | 46 | nation | 80 | for | 114 | we | 148 | what | 182 | thus | 216 | which | 250 | freedom |  |
| 13 | continent | 47 | so | 81 | those | 115 | cannot | 149 | we | 183 | far | 217 | they | 251 | and |  |
| 14 | a | 48 | conceived | 82 | who | 116 | hallow | 150 | say | 184 | so | 218 | gave | 252 | that |  |
| 15 | new | 49 | and | 83 | here | 117 | this | 151 | here | 185 | nobly | 219 | the | 253 | government |  |
| 16 | nation | 50 | so | 84 | gave | 118 | ground | 152 | but | 186 | advanced | 220 | last | 254 | of |  |
| 17 | conceived | 51 | dedicated | 85 | their | 119 | The | 153 | it | 187 | It | 221 | full | 255 | the |  |
| 18 | in | 52 | can | 86 | lives | 120 | brave | 154 | can | 188 | is | 222 | measure | 256 | people |  |
| 19 | liberty | 53 | long | 87 | that | 121 | men | 155 | never | 189 | rather | 223 | of | 257 | by |  |
| 20 | and | 54 | endure | 88 | that | 122 | living | 156 | forget | 190 | for | 224 | devotion | 258 | the |  |
| 21 | dedicated | 55 | We | 89 | nation | 123 | and | 157 | what | 191 | us | 225 | that | 259 | people |  |
| 22 | to | 56 | are | 90 | might | 124 | dead | 158 | they | 192 | to | 226 | we | 260 | for |  |
| 23 | the | 57 | met | 91 | live | 125 | who | 159 | did | 193 | be | 227 | here | 261 | the |  |
| 24 | proposition | 58 | on | 92 | It | 126 | struggled | 160 | here | 194 | here | 228 | highly | 262 | people |  |
| 25 | that | 59 | a | 93 | is | 127 | here | 161 | It | 195 | dedicated | 229 | resolve | 263 | shall |  |
| 26 | all | 60 | great | 94 | altogether | 128 | have | 162 | is | 196 | to | 230 | that | 264 | not |  |
| 27 | men | 61 | battlefield | 95 | fitting | 129 | consecrated | 163 | for | 197 | the | 231 | these | 265 | perish |  |
| 28 | are | 62 | of | 96 | and | 130 | it | 164 | us | 198 | great | 232 | dead | 266 | from |  |
| 29 | created | 63 | that | 97 | proper | 131 | far | 165 | the | 199 | task | 233 | shall | 267 | the |  |
| 30 | equal | 64 | war | 98 | that | 132 | above | 166 | living | 200 | remaining | 234 | not | 268 | earth |  |
| 31 | Now | 65 | We | 99 | we | 133 | our | 167 | rather | 201 | before | 235 | have |  |  |  |
| 32 | we | 66 | have | 100 | should | 134 | poor | 168 | to | 202 | us | 236 | died |  |  |  |
| 33 | are | 67 | come | 101 | do | 135 | power | 169 | be | 203 | that | 237 | in |  |  |  |
| 34 | engaged | 68 | to | 102 | this | 136 | to | 170 | dedicated | 204 | from | 238 | vain |  |  |  |

**Random Digits Table**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Row |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 4 | 2 | 0 | 4 | 1 |  | 7 | 9 | 0 | 0 | 6 |  | 3 | 3 | 4 | 2 | 9 |  | 0 | 9 | 5 | 1 | 8 |  | 1 | 6 | 4 | 5 | 3 |  | 6 | 2 | 7 | 0 | 4 |  | 8 | 2 | 3 | 4 | 0 |  |
| 2 | 7 | 8 | 8 | 8 | 6 |  | 5 | 7 | 0 | 6 | 5 |  | 2 | 8 | 6 | 9 | 3 |  | 9 | 4 | 5 | 5 | 7 |  | 0 | 9 | 0 | 9 | 9 |  | 2 | 0 | 3 | 4 | 3 |  | 3 | 2 | 2 | 7 | 6 |  |
| 3 | 5 | 5 | 9 | 2 | 7 |  | 4 | 8 | 0 | 0 | 7 |  | 1 | 7 | 0 | 6 | 0 |  | 5 | 5 | 4 | 8 | 2 |  | 6 | 8 | 1 | 5 | 0 |  | 0 | 5 | 5 | 2 | 3 |  | 5 | 0 | 2 | 6 | 5 |  |
| 4 | 9 | 3 | 7 | 5 | 1 |  | 8 | 1 | 5 | 9 | 6 |  | 1 | 9 | 4 | 8 | 6 |  | 5 | 1 | 2 | 5 | 1 |  | 4 | 6 | 2 | 8 | 3 |  | 3 | 2 | 6 | 6 | 9 |  | 5 | 7 | 9 | 7 | 3 |  |
| 5 | 6 | 5 | 7 | 5 | 6 |  | 9 | 0 | 8 | 1 | 7 |  | 9 | 7 | 8 | 9 | 5 |  | 3 | 5 | 9 | 6 | 7 |  | 8 | 1 | 5 | 5 | 5 |  | 8 | 8 | 1 | 0 | 1 |  | 8 | 4 | 7 | 5 | 1 |  |
| 6 | 1 | 9 | 8 | 8 | 7 |  | 5 | 8 | 7 | 4 | 2 |  | 7 | 4 | 0 | 2 | 1 |  | 1 | 9 | 3 | 5 | 9 |  | 7 | 4 | 7 | 5 | 1 |  | 1 | 5 | 4 | 1 | 4 |  | 8 | 2 | 6 | 1 | 7 |  |
| 7 | 3 | 8 | 5 | 3 | 6 |  | 2 | 8 | 3 | 1 | 2 |  | 0 | 1 | 6 | 0 | 0 |  | 3 | 3 | 9 | 7 | 6 |  | 7 | 8 | 0 | 2 | 8 |  | 6 | 1 | 6 | 4 | 0 |  | 0 | 1 | 2 | 4 | 1 |  |
| 8 | 8 | 9 | 7 | 0 | 2 |  | 9 | 1 | 5 | 2 | 1 |  | 0 | 1 | 2 | 1 | 3 |  | 5 | 2 | 1 | 3 | 0 |  | 9 | 0 | 6 | 5 | 0 |  | 9 | 4 | 3 | 3 | 1 |  | 0 | 6 | 6 | 1 | 2 |  |
| 9 | 9 | 3 | 8 | 7 | 3 |  | 4 | 0 | 5 | 0 | 8 |  | 3 | 4 | 5 | 5 | 9 |  | 5 | 3 | 1 | 2 | 8 |  | 1 | 4 | 3 | 0 | 9 |  | 4 | 8 | 7 | 9 | 4 |  | 7 | 6 | 0 | 5 | 9 |  |
| 10 | 2 | 7 | 0 | 0 | 4 |  | 4 | 3 | 2 | 3 | 6 |  | 7 | 3 | 0 | 2 | 6 |  | 4 | 3 | 7 | 6 | 1 |  | 1 | 6 | 1 | 2 | 1 |  | 1 | 7 | 9 | 9 | 0 |  | 9 | 5 | 8 | 0 | 4 |  |
| 11 | 1 | 3 | 8 | 7 | 4 |  | 1 | 9 | 8 | 7 | 4 |  | 5 | 2 | 1 | 4 | 1 |  | 7 | 0 | 8 | 0 | 4 |  | 7 | 5 | 4 | 2 | 5 |  | 2 | 9 | 2 | 1 | 4 |  | 7 | 7 | 7 | 9 | 3 |  |
| 12 | 2 | 7 | 9 | 3 | 4 |  | 1 | 5 | 5 | 6 | 4 |  | 3 | 6 | 9 | 5 | 4 |  | 0 | 3 | 6 | 7 | 2 |  | 6 | 6 | 5 | 2 | 0 |  | 8 | 4 | 2 | 0 | 4 |  | 1 | 0 | 5 | 6 | 0 |  |
| 13 | 4 | 2 | 8 | 8 | 5 |  | 1 | 8 | 4 | 3 | 7 |  | 0 | 9 | 9 | 2 | 3 |  | 2 | 0 | 4 | 0 | 6 |  | 8 | 3 | 0 | 1 | 1 |  | 7 | 6 | 1 | 0 | 0 |  | 3 | 7 | 5 | 5 | 1 |  |
| 14 | 0 | 5 | 6 | 7 | 7 |  | 0 | 0 | 7 | 8 | 0 |  | 3 | 9 | 0 | 2 | 7 |  | 7 | 8 | 6 | 1 | 0 |  | 6 | 8 | 0 | 8 | 2 |  | 3 | 2 | 0 | 6 | 1 |  | 0 | 7 | 6 | 1 | 7 |  |
| 15 | 3 | 1 | 4 | 8 | 8 |  | 1 | 9 | 4 | 8 | 1 |  | 0 | 0 | 3 | 9 | 5 |  | 9 | 9 | 0 | 9 | 4 |  | 2 | 8 | 1 | 0 | 9 |  | 0 | 0 | 7 | 3 | 9 |  | 5 | 7 | 1 | 7 | 3 |  |
| 16 | 9 | 1 | 8 | 1 | 5 |  | 0 | 4 | 4 | 3 | 9 |  | 5 | 8 | 0 | 7 | 6 |  | 2 | 3 | 1 | 7 | 2 |  | 9 | 5 | 1 | 6 | 3 |  | 6 | 3 | 9 | 5 | 9 |  | 5 | 3 | 2 | 0 | 1 |  |
| 17 | 0 | 8 | 2 | 1 | 6 |  | 7 | 2 | 2 | 6 | 8 |  | 6 | 5 | 1 | 8 | 6 |  | 2 | 2 | 3 | 6 | 8 |  | 1 | 2 | 7 | 2 | 4 |  | 6 | 5 | 4 | 9 | 8 |  | 5 | 0 | 3 | 3 | 5 |  |
| 18 | 2 | 6 | 9 | 1 | 7 |  | 2 | 8 | 9 | 2 | 8 |  | 2 | 4 | 0 | 3 | 9 |  | 9 | 5 | 9 | 8 | 8 |  | 6 | 1 | 5 | 1 | 8 |  | 9 | 1 | 3 | 3 | 3 |  | 7 | 9 | 2 | 8 | 2 |  |
| 19 | 1 | 3 | 3 | 2 | 1 |  | 0 | 5 | 1 | 2 | 1 |  | 9 | 8 | 0 | 2 | 4 |  | 3 | 5 | 4 | 8 | 5 |  | 6 | 2 | 2 | 6 | 9 |  | 3 | 5 | 7 | 0 | 9 |  | 6 | 5 | 1 | 6 | 7 |  |
| 20 | 9 | 7 | 1 | 4 | 0 |  | 2 | 8 | 4 | 3 | 3 |  | 6 | 3 | 3 | 8 | 1 |  | 9 | 9 | 7 | 5 | 6 |  | 3 | 5 | 1 | 5 | 5 |  | 2 | 9 | 4 | 0 | 7 |  | 1 | 6 | 9 | 6 | 8 |  |
| 21 | 7 | 6 | 6 | 0 | 5 |  | 1 | 8 | 7 | 6 | 8 |  | 2 | 9 | 1 | 7 | 6 |  | 0 | 3 | 9 | 8 | 7 |  | 2 | 5 | 6 | 9 | 1 |  | 1 | 8 | 9 | 5 | 4 |  | 9 | 0 | 3 | 6 | 5 |  |
| 22 | 4 | 0 | 8 | 0 | 9 |  | 9 | 2 | 2 | 1 | 4 |  | 4 | 4 | 1 | 4 | 7 |  | 7 | 3 | 9 | 1 | 1 |  | 7 | 6 | 8 | 8 | 3 |  | 2 | 6 | 3 | 9 | 9 |  | 7 | 4 | 6 | 9 | 8 |  |
| 23 | 6 | 8 | 1 | 6 | 5 |  | 1 | 3 | 0 | 0 | 4 |  | 0 | 4 | 1 | 0 | 5 |  | 0 | 1 | 8 | 7 | 0 |  | 4 | 1 | 0 | 0 | 5 |  | 7 | 7 | 1 | 7 | 1 |  | 0 | 1 | 5 | 6 | 5 |  |
| 24 | 0 | 5 | 9 | 5 | 3 |  | 2 | 4 | 7 | 0 | 7 |  | 7 | 8 | 0 | 8 | 9 |  | 2 | 0 | 1 | 8 | 1 |  | 7 | 0 | 9 | 2 | 3 |  | 0 | 3 | 5 | 5 | 9 |  | 0 | 4 | 3 | 4 | 3 |  |
| 25 | 5 | 3 | 5 | 7 | 0 |  | 5 | 4 | 0 | 0 | 3 |  | 1 | 5 | 3 | 3 | 8 |  | 2 | 0 | 8 | 3 | 3 |  | 0 | 0 | 9 | 0 | 8 |  | 2 | 3 | 3 | 3 | 2 |  | 1 | 8 | 0 | 7 | 1 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 26 | 2 | 1 | 2 | 4 | 4 |  | 5 | 7 | 3 | 3 | 1 |  | 5 | 0 | 6 | 1 | 9 |  | 1 | 3 | 9 | 7 | 7 |  | 4 | 8 | 5 | 7 | 5 |  | 2 | 2 | 6 | 5 | 4 |  | 8 | 2 | 1 | 8 | 8 |  |
| 27 | 1 | 9 | 0 | 3 | 4 |  | 6 | 0 | 4 | 7 | 5 |  | 7 | 4 | 9 | 5 | 3 |  | 6 | 3 | 0 | 4 | 8 |  | 4 | 0 | 9 | 9 | 7 |  | 2 | 7 | 6 | 5 | 1 |  | 5 | 6 | 6 | 1 | 3 |  |
| 28 | 5 | 8 | 8 | 9 | 6 |  | 9 | 0 | 3 | 0 | 7 |  | 0 | 7 | 3 | 2 | 2 |  | 2 | 0 | 2 | 6 | 2 |  | 1 | 1 | 6 | 4 | 1 |  | 8 | 7 | 6 | 8 | 0 |  | 3 | 5 | 6 | 9 | 9 |  |
| 29 | 5 | 2 | 7 | 0 | 7 |  | 8 | 7 | 3 | 5 | 0 |  | 1 | 0 | 5 | 1 | 5 |  | 6 | 9 | 9 | 1 | 5 |  | 5 | 4 | 9 | 3 | 4 |  | 0 | 9 | 0 | 8 | 0 |  | 8 | 4 | 4 | 2 | 6 |  |
| 30 | 2 | 8 | 3 | 1 | 1 |  | 8 | 0 | 6 | 1 | 6 |  | 5 | 7 | 7 | 9 | 7 |  | 4 | 2 | 3 | 4 | 4 |  | 9 | 7 | 2 | 0 | 6 |  | 4 | 4 | 4 | 9 | 7 |  | 5 | 3 | 1 | 6 | 4 |  |
| 31 | 2 | 6 | 2 | 3 | 3 |  | 3 | 4 | 6 | 3 | 2 |  | 7 | 9 | 4 | 4 | 0 |  | 6 | 0 | 0 | 0 | 2 |  | 5 | 5 | 2 | 7 | 0 |  | 0 | 8 | 9 | 1 | 8 |  | 4 | 8 | 7 | 7 | 9 |  |
| 32 | 6 | 4 | 7 | 4 | 2 |  | 7 | 0 | 2 | 5 | 7 |  | 6 | 4 | 3 | 3 | 9 |  | 0 | 8 | 2 | 7 | 3 |  | 5 | 6 | 6 | 9 | 6 |  | 7 | 8 | 7 | 0 | 5 |  | 5 | 4 | 8 | 8 | 4 |  |
| 33 | 8 | 6 | 3 | 1 | 8 |  | 0 | 5 | 3 | 6 | 8 |  | 5 | 1 | 4 | 4 | 9 |  | 9 | 0 | 6 | 2 | 4 |  | 9 | 7 | 7 | 1 | 8 |  | 0 | 4 | 0 | 6 | 8 |  | 8 | 4 | 2 | 9 | 7 |  |
| 34 | 8 | 6 | 3 | 7 | 3 |  | 7 | 9 | 7 | 9 | 1 |  | 2 | 0 | 3 | 3 | 0 |  | 0 | 7 | 6 | 5 | 9 |  | 7 | 4 | 8 | 4 | 9 |  | 5 | 5 | 1 | 2 | 4 |  | 5 | 4 | 4 | 0 | 0 |  |
| 35 | 2 | 8 | 6 | 8 | 7 |  | 2 | 8 | 8 | 0 | 4 |  | 6 | 1 | 7 | 2 | 8 |  | 5 | 1 | 6 | 3 | 4 |  | 9 | 2 | 9 | 4 | 9 |  | 9 | 8 | 9 | 4 | 4 |  | 8 | 2 | 7 | 0 | 0 |  |
| 36 | 6 | 1 | 2 | 5 | 6 |  | 1 | 0 | 6 | 1 | 7 |  | 1 | 1 | 7 | 7 | 8 |  | 3 | 0 | 5 | 8 | 0 |  | 9 | 5 | 7 | 8 | 7 |  | 8 | 3 | 7 | 6 | 6 |  | 6 | 8 | 9 | 4 | 0 |  |
| 37 | 5 | 6 | 4 | 5 | 4 |  | 8 | 6 | 7 | 3 | 6 |  | 9 | 5 | 5 | 2 | 2 |  | 3 | 5 | 4 | 8 | 7 |  | 7 | 8 | 1 | 3 | 6 |  | 0 | 2 | 2 | 3 | 0 |  | 8 | 0 | 7 | 0 | 2 |  |
| 38 | 5 | 0 | 2 | 8 | 2 |  | 4 | 9 | 3 | 9 | 2 |  | 4 | 5 | 8 | 6 | 9 |  | 1 | 1 | 2 | 0 | 8 |  | 8 | 1 | 5 | 0 | 3 |  | 0 | 6 | 6 | 5 | 8 |  | 6 | 8 | 3 | 3 | 0 |  |
| 39 | 3 | 3 | 2 | 2 | 7 |  | 7 | 5 | 5 | 4 | 7 |  | 3 | 3 | 5 | 3 | 8 |  | 9 | 5 | 0 | 4 | 9 |  | 8 | 6 | 1 | 9 | 0 |  | 1 | 6 | 5 | 4 | 0 |  | 7 | 8 | 4 | 2 | 7 |  |
| 40 | 6 | 2 | 4 | 0 | 0 |  | 2 | 2 | 5 | 3 | 8 |  | 1 | 6 | 7 | 2 | 0 |  | 0 | 7 | 2 | 9 | 9 |  | 2 | 4 | 1 | 0 | 0 |  | 3 | 9 | 6 | 6 | 2 |  | 8 | 8 | 1 | 1 | 2 |  |
| 41 | 4 | 8 | 4 | 2 | 4 |  | 7 | 5 | 4 | 4 | 0 |  | 7 | 5 | 5 | 9 | 5 |  | 3 | 9 | 2 | 8 | 3 |  | 0 | 2 | 9 | 7 | 5 |  | 3 | 7 | 9 | 6 | 9 |  | 0 | 4 | 6 | 7 | 3 |  |
| 42 | 7 | 2 | 4 | 3 | 4 |  | 5 | 8 | 9 | 2 | 4 |  | 3 | 1 | 2 | 4 | 0 |  | 4 | 1 | 4 | 1 | 8 |  | 0 | 0 | 4 | 5 | 2 |  | 6 | 1 | 4 | 1 | 4 |  | 3 | 5 | 7 | 6 | 3 |  |
| 43 | 8 | 6 | 2 | 0 | 1 |  | 9 | 4 | 9 | 1 | 1 |  | 7 | 5 | 5 | 4 | 0 |  | 4 | 6 | 3 | 3 | 4 |  | 5 | 3 | 5 | 6 | 2 |  | 4 | 5 | 1 | 7 | 1 |  | 9 | 4 | 8 | 5 | 8 |  |
| 44 | 0 | 0 | 4 | 7 | 6 |  | 3 | 2 | 0 | 5 | 4 |  | 2 | 5 | 6 | 0 | 7 |  | 3 | 2 | 0 | 2 | 8 |  | 0 | 6 | 7 | 9 | 6 |  | 0 | 8 | 2 | 8 | 6 |  | 7 | 1 | 8 | 8 | 4 |  |
| 45 | 0 | 6 | 4 | 9 | 9 |  | 7 | 6 | 8 | 0 | 0 |  | 6 | 7 | 6 | 4 | 4 |  | 5 | 7 | 8 | 1 | 1 |  | 6 | 3 | 3 | 9 | 3 |  | 6 | 6 | 0 | 8 | 8 |  | 2 | 8 | 3 | 1 | 8 |  |
| 46 | 1 | 3 | 7 | 6 | 2 |  | 4 | 9 | 3 | 3 | 8 |  | 3 | 6 | 4 | 0 | 2 |  | 4 | 3 | 4 | 1 | 4 |  | 1 | 5 | 2 | 3 | 0 |  | 8 | 9 | 3 | 4 | 5 |  | 2 | 3 | 8 | 6 | 8 |  |
| 47 | 9 | 3 | 5 | 1 | 5 |  | 9 | 1 | 0 | 3 | 3 |  | 9 | 9 | 5 | 5 | 4 |  | 7 | 4 | 1 | 6 | 0 |  | 8 | 7 | 1 | 7 | 2 |  | 8 | 7 | 3 | 8 | 4 |  | 6 | 3 | 3 | 3 | 1 |  |
| 48 | 3 | 3 | 8 | 7 | 8 |  | 4 | 2 | 1 | 0 | 1 |  | 7 | 7 | 6 | 0 | 0 |  | 4 | 2 | 4 | 6 | 6 |  | 0 | 3 | 9 | 1 | 3 |  | 6 | 0 | 6 | 8 | 6 |  | 6 | 6 | 4 | 3 | 1 |  |
| 49 | 9 | 1 | 1 | 7 | 2 |  | 5 | 9 | 5 | 6 | 8 |  | 6 | 9 | 6 | 2 | 1 |  | 5 | 8 | 1 | 5 | 0 |  | 9 | 9 | 7 | 0 | 9 |  | 4 | 5 | 9 | 3 | 5 |  | 7 | 5 | 2 | 0 | 3 |  |
| 50 | 3 | 2 | 8 | 9 | 5 |  | 0 | 5 | 0 | 6 | 0 |  | 0 | 6 | 7 | 2 | 5 |  | 4 | 9 | 6 | 6 | 4 |  | 1 | 6 | 9 | 5 | 9 |  | 7 | 5 | 8 | 6 | 4 |  | 5 | 2 | 0 | 6 | 6 |  |

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| 51 | 3 | 7 | 0 | 8 | 3 |  | 5 | 3 | 4 | 7 | 0 |  | 6 | 1 | 3 | 2 | 9 |  | 2 | 6 | 0 | 4 | 1 |  | 6 | 7 | 6 | 0 | 1 |  | 4 | 3 | 8 | 1 | 0 |  | 3 | 1 | 8 | 8 | 6 |  |
| 52 | 4 | 1 | 3 | 7 | 7 |  | 0 | 2 | 8 | 4 | 1 |  | 3 | 3 | 9 | 6 | 1 |  | 3 | 6 | 5 | 5 | 2 |  | 6 | 8 | 8 | 0 | 5 |  | 9 | 3 | 7 | 3 | 7 |  | 9 | 7 | 0 | 9 | 9 |  |
| 53 | 0 | 8 | 6 | 5 | 9 |  | 7 | 9 | 2 | 0 | 6 |  | 0 | 2 | 7 | 8 | 3 |  | 1 | 7 | 4 | 2 | 8 |  | 7 | 4 | 6 | 0 | 7 |  | 3 | 2 | 5 | 7 | 5 |  | 7 | 5 | 0 | 7 | 4 |  |
| 54 | 0 | 8 | 9 | 2 | 7 |  | 8 | 9 | 9 | 8 | 3 |  | 5 | 3 | 2 | 9 | 7 |  | 2 | 1 | 7 | 7 | 8 |  | 1 | 6 | 1 | 1 | 5 |  | 4 | 4 | 0 | 1 | 3 |  | 6 | 1 | 5 | 9 | 4 |  |
| 55 | 6 | 8 | 9 | 1 | 6 |  | 5 | 6 | 5 | 2 | 4 |  | 3 | 2 | 4 | 6 | 3 |  | 3 | 3 | 1 | 5 | 2 |  | 0 | 9 | 2 | 9 | 0 |  | 9 | 4 | 5 | 4 | 1 |  | 1 | 9 | 4 | 4 | 1 |  |
| 56 | 2 | 8 | 0 | 5 | 1 |  | 9 | 1 | 4 | 6 | 6 |  | 6 | 9 | 3 | 3 | 8 |  | 7 | 0 | 6 | 2 | 9 |  | 5 | 0 | 5 | 8 | 8 |  | 1 | 0 | 3 | 4 | 6 |  | 1 | 0 | 5 | 6 | 9 |  |
| 57 | 9 | 5 | 3 | 1 | 6 |  | 5 | 8 | 8 | 4 | 6 |  | 5 | 4 | 4 | 4 | 4 |  | 6 | 3 | 2 | 8 | 9 |  | 3 | 5 | 9 | 6 | 6 |  | 0 | 0 | 0 | 7 | 0 |  | 2 | 0 | 5 | 3 | 5 |  |
| 58 | 1 | 3 | 6 | 2 | 3 |  | 1 | 4 | 7 | 6 | 7 |  | 2 | 0 | 0 | 4 | 1 |  | 0 | 6 | 8 | 4 | 6 |  | 8 | 1 | 2 | 7 | 1 |  | 7 | 5 | 8 | 4 | 2 |  | 5 | 8 | 8 | 3 | 0 |  |
| 59 | 7 | 3 | 8 | 4 | 1 |  | 8 | 5 | 9 | 4 | 9 |  | 6 | 8 | 5 | 4 | 8 |  | 1 | 6 | 9 | 8 | 5 |  | 8 | 6 | 7 | 6 | 1 |  | 0 | 9 | 0 | 7 | 0 |  | 5 | 2 | 7 | 1 | 9 |  |
| 60 | 2 | 5 | 3 | 9 | 4 |  | 9 | 3 | 1 | 7 | 3 |  | 6 | 3 | 7 | 4 | 2 |  | 4 | 9 | 8 | 5 | 1 |  | 0 | 1 | 9 | 8 | 5 |  | 6 | 7 | 2 | 8 | 7 |  | 7 | 3 | 0 | 5 | 6 |  |
| 61 | 5 | 0 | 2 | 7 | 2 |  | 3 | 5 | 9 | 9 | 2 |  | 5 | 8 | 9 | 6 | 6 |  | 0 | 0 | 5 | 6 | 8 |  | 1 | 3 | 4 | 5 | 6 |  | 1 | 7 | 2 | 4 | 6 |  | 0 | 4 | 2 | 2 | 4 |  |
| 62 | 4 | 5 | 1 | 0 | 5 |  | 1 | 0 | 7 | 7 | 4 |  | 8 | 8 | 2 | 4 | 1 |  | 1 | 9 | 9 | 3 | 1 |  | 5 | 6 | 4 | 7 | 5 |  | 9 | 0 | 0 | 8 | 7 |  | 4 | 2 | 3 | 1 | 1 |  |
| 63 | 2 | 9 | 6 | 4 | 2 |  | 7 | 6 | 5 | 8 | 1 |  | 8 | 1 | 8 | 2 | 8 |  | 7 | 6 | 2 | 7 | 5 |  | 5 | 4 | 9 | 8 | 5 |  | 9 | 4 | 0 | 3 | 7 |  | 8 | 7 | 3 | 4 | 1 |  |
| 64 | 9 | 7 | 8 | 3 | 4 |  | 0 | 0 | 2 | 1 | 5 |  | 2 | 9 | 8 | 0 | 2 |  | 7 | 9 | 1 | 1 | 7 |  | 8 | 6 | 3 | 4 | 9 |  | 9 | 4 | 9 | 3 | 5 |  | 0 | 0 | 8 | 9 | 9 |  |
| 65 | 3 | 3 | 5 | 6 | 6 |  | 6 | 9 | 1 | 0 | 8 |  | 2 | 2 | 8 | 5 | 7 |  | 7 | 3 | 4 | 3 | 9 |  | 0 | 2 | 4 | 2 | 4 |  | 5 | 5 | 4 | 0 | 0 |  | 0 | 3 | 4 | 3 | 3 |  |
| 66 | 6 | 5 | 1 | 2 | 6 |  | 2 | 2 | 6 | 9 | 4 |  | 3 | 9 | 5 | 1 | 5 |  | 5 | 7 | 8 | 8 | 9 |  | 6 | 8 | 8 | 2 | 3 |  | 1 | 7 | 7 | 7 | 0 |  | 6 | 3 | 8 | 1 | 1 |  |
| 67 | 8 | 4 | 6 | 8 | 5 |  | 6 | 7 | 0 | 9 | 9 |  | 0 | 9 | 1 | 8 | 0 |  | 8 | 6 | 2 | 6 | 6 |  | 0 | 9 | 6 | 4 | 8 |  | 8 | 4 | 4 | 0 | 7 |  | 2 | 3 | 7 | 7 | 7 |  |
| 68 | 9 | 0 | 7 | 1 | 6 |  | 1 | 7 | 7 | 4 | 4 |  | 3 | 2 | 8 | 5 | 7 |  | 5 | 7 | 5 | 3 | 7 |  | 5 | 7 | 9 | 4 | 3 |  | 6 | 8 | 1 | 3 | 6 |  | 2 | 3 | 9 | 4 | 6 |  |
| 69 | 6 | 3 | 7 | 6 | 9 |  | 5 | 6 | 4 | 1 | 2 |  | 1 | 7 | 9 | 0 | 9 |  | 1 | 6 | 1 | 3 | 4 |  | 5 | 8 | 1 | 7 | 7 |  | 9 | 5 | 9 | 9 | 7 |  | 3 | 6 | 4 | 6 | 8 |  |
| 70 | 2 | 2 | 8 | 9 | 5 |  | 8 | 2 | 3 | 1 | 6 |  | 7 | 4 | 1 | 2 | 9 |  | 6 | 0 | 4 | 5 | 1 |  | 9 | 6 | 8 | 6 | 6 |  | 5 | 9 | 0 | 2 | 4 |  | 2 | 8 | 9 | 0 | 1 |  |
| 71 | 3 | 2 | 7 | 5 | 8 |  | 9 | 6 | 1 | 6 | 5 |  | 3 | 7 | 2 | 7 | 8 |  | 6 | 1 | 8 | 9 | 0 |  | 2 | 9 | 3 | 9 | 9 |  | 2 | 4 | 8 | 0 | 7 |  | 5 | 8 | 1 | 8 | 4 |  |
| 72 | 9 | 1 | 8 | 6 | 8 |  | 8 | 0 | 8 | 2 | 4 |  | 1 | 5 | 0 | 6 | 4 |  | 8 | 1 | 5 | 2 | 3 |  | 6 | 9 | 8 | 1 | 3 |  | 0 | 8 | 5 | 2 | 0 |  | 0 | 0 | 4 | 1 | 9 |  |
| 73 | 9 | 2 | 1 | 0 | 7 |  | 8 | 7 | 5 | 3 | 8 |  | 4 | 3 | 4 | 6 | 7 |  | 1 | 3 | 5 | 1 | 2 |  | 0 | 5 | 2 | 9 | 3 |  | 9 | 9 | 7 | 2 | 3 |  | 7 | 5 | 9 | 8 | 0 |  |
| 74 | 9 | 1 | 5 | 4 | 4 |  | 9 | 5 | 6 | 4 | 6 |  | 8 | 8 | 6 | 0 | 8 |  | 0 | 0 | 5 | 9 | 3 |  | 8 | 8 | 5 | 7 | 0 |  | 3 | 1 | 0 | 8 | 9 |  | 8 | 2 | 1 | 6 | 3 |  |
| 75 | 7 | 5 | 7 | 3 | 4 |  | 3 | 2 | 5 | 9 | 7 |  | 0 | 1 | 6 | 8 | 5 |  | 4 | 2 | 7 | 7 | 8 |  | 1 | 5 | 6 | 2 | 2 |  | 3 | 1 | 2 | 9 | 2 |  | 1 | 8 | 9 | 8 | 9 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| 76 | 2 | 4 | 6 | 8 | 2 |  | 8 | 6 | 9 | 3 | 3 |  | 9 | 5 | 9 | 0 | 5 |  | 8 | 7 | 2 | 0 | 5 |  | 1 | 6 | 7 | 3 | 0 |  | 1 | 9 | 8 | 0 | 1 |  | 1 | 3 | 6 | 0 | 5 |  |
| 77 | 2 | 7 | 0 | 6 | 9 |  | 6 | 7 | 6 | 5 | 3 |  | 0 | 4 | 8 | 9 | 9 |  | 2 | 6 | 1 | 8 | 6 |  | 7 | 8 | 1 | 7 | 0 |  | 1 | 2 | 9 | 0 | 7 |  | 3 | 5 | 4 | 2 | 3 |  |
| 78 | 2 | 7 | 2 | 6 | 2 |  | 4 | 5 | 4 | 8 | 7 |  | 4 | 2 | 4 | 9 | 3 |  | 6 | 9 | 4 | 7 | 2 |  | 9 | 7 | 3 | 3 | 6 |  | 8 | 5 | 0 | 0 | 1 |  | 4 | 0 | 4 | 7 | 4 |  |
| 79 | 7 | 3 | 7 | 9 | 1 |  | 9 | 3 | 9 | 2 | 0 |  | 1 | 3 | 6 | 5 | 4 |  | 4 | 0 | 6 | 1 | 6 |  | 4 | 3 | 8 | 1 | 9 |  | 6 | 0 | 8 | 9 | 5 |  | 0 | 4 | 3 | 6 | 3 |  |
| 80 | 2 | 6 | 4 | 3 | 6 |  | 8 | 4 | 5 | 9 | 1 |  | 1 | 5 | 1 | 5 | 9 |  | 7 | 6 | 0 | 5 | 0 |  | 9 | 4 | 8 | 9 | 4 |  | 2 | 0 | 3 | 5 | 2 |  | 2 | 3 | 5 | 8 | 6 |  |
| 81 | 1 | 0 | 9 | 4 | 2 |  | 0 | 8 | 7 | 0 | 8 |  | 6 | 9 | 0 | 6 | 0 |  | 6 | 3 | 0 | 0 | 6 |  | 4 | 4 | 3 | 5 | 1 |  | 4 | 6 | 1 | 7 | 5 |  | 5 | 1 | 6 | 4 | 8 |  |
| 82 | 6 | 0 | 0 | 4 | 0 |  | 8 | 2 | 2 | 4 | 5 |  | 2 | 0 | 2 | 3 | 2 |  | 1 | 7 | 7 | 4 | 1 |  | 0 | 7 | 6 | 2 | 2 |  | 0 | 3 | 6 | 5 | 4 |  | 9 | 9 | 1 | 9 | 8 |  |
| 83 | 8 | 1 | 9 | 9 | 5 |  | 2 | 3 | 3 | 2 | 5 |  | 8 | 8 | 1 | 7 | 8 |  | 0 | 1 | 7 | 3 | 2 |  | 9 | 5 | 3 | 6 | 8 |  | 4 | 7 | 2 | 0 | 4 |  | 4 | 7 | 6 | 6 | 8 |  |
| 84 | 4 | 0 | 0 | 6 | 6 |  | 2 | 1 | 5 | 1 | 6 |  | 0 | 3 | 7 | 1 | 6 |  | 4 | 1 | 1 | 3 | 3 |  | 4 | 3 | 9 | 8 | 0 |  | 0 | 5 | 9 | 3 | 7 |  | 1 | 0 | 4 | 9 | 9 |  |
| 85 | 0 | 7 | 6 | 7 | 8 |  | 7 | 6 | 3 | 0 | 3 |  | 3 | 4 | 1 | 6 | 4 |  | 7 | 7 | 7 | 4 | 1 |  | 5 | 6 | 2 | 4 | 6 |  | 4 | 5 | 1 | 7 | 6 |  | 2 | 6 | 6 | 3 | 7 |  |
| 86 | 5 | 6 | 9 | 2 | 9 |  | 9 | 7 | 4 | 7 | 2 |  | 1 | 6 | 2 | 3 | 3 |  | 2 | 0 | 7 | 5 | 8 |  | 2 | 1 | 9 | 2 | 5 |  | 4 | 0 | 7 | 3 | 9 |  | 0 | 9 | 6 | 9 | 4 |  |
| 87 | 0 | 3 | 7 | 5 | 0 |  | 8 | 4 | 9 | 4 | 6 |  | 6 | 7 | 5 | 8 | 2 |  | 6 | 9 | 9 | 1 | 7 |  | 1 | 6 | 2 | 4 | 7 |  | 8 | 6 | 4 | 5 | 5 |  | 5 | 6 | 9 | 1 | 6 |  |
| 88 | 9 | 1 | 7 | 7 | 0 |  | 3 | 6 | 2 | 7 | 3 |  | 7 | 5 | 5 | 2 | 3 |  | 7 | 7 | 7 | 4 | 9 |  | 9 | 0 | 9 | 2 | 4 |  | 5 | 5 | 4 | 6 | 4 |  | 7 | 8 | 0 | 3 | 9 |  |
| 89 | 5 | 3 | 1 | 6 | 9 |  | 0 | 0 | 8 | 4 | 9 |  | 6 | 0 | 8 | 5 | 1 |  | 5 | 3 | 2 | 3 | 9 |  | 1 | 7 | 5 | 1 | 1 |  | 1 | 1 | 3 | 0 | 9 |  | 8 | 4 | 5 | 5 | 4 |  |
| 90 | 9 | 7 | 6 | 4 | 6 |  | 8 | 8 | 3 | 6 | 1 |  | 6 | 1 | 7 | 1 | 9 |  | 3 | 0 | 1 | 5 | 3 |  | 8 | 8 | 3 | 7 | 3 |  | 5 | 6 | 2 | 3 | 0 |  | 4 | 8 | 8 | 6 | 4 |  |
| 91 | 0 | 5 | 9 | 0 | 4 |  | 2 | 4 | 4 | 9 | 2 |  | 9 | 7 | 8 | 1 | 9 |  | 3 | 3 | 5 | 6 | 1 |  | 2 | 1 | 0 | 8 | 2 |  | 9 | 9 | 1 | 8 | 6 |  | 9 | 6 | 3 | 8 | 7 |  |
| 92 | 2 | 7 | 8 | 3 | 0 |  | 0 | 6 | 3 | 4 | 7 |  | 5 | 0 | 9 | 9 | 0 |  | 7 | 9 | 5 | 6 | 2 |  | 4 | 9 | 1 | 6 | 3 |  | 7 | 1 | 4 | 3 | 9 |  | 3 | 7 | 5 | 7 | 2 |  |
| 93 | 7 | 6 | 7 | 6 | 8 |  | 6 | 3 | 0 | 8 | 0 |  | 2 | 9 | 0 | 9 | 6 |  | 7 | 6 | 5 | 5 | 9 |  | 3 | 4 | 0 | 3 | 1 |  | 5 | 6 | 4 | 4 | 1 |  | 6 | 5 | 9 | 0 | 8 |  |
| 94 | 4 | 7 | 7 | 1 | 4 |  | 9 | 5 | 4 | 1 | 9 |  | 1 | 5 | 8 | 7 | 3 |  | 8 | 3 | 6 | 6 | 0 |  | 3 | 5 | 1 | 8 | 2 |  | 6 | 7 | 9 | 4 | 3 |  | 1 | 4 | 1 | 9 | 2 |  |
| 95 | 7 | 4 | 8 | 0 | 4 |  | 7 | 0 | 9 | 9 | 3 |  | 7 | 7 | 2 | 9 | 0 |  | 2 | 7 | 6 | 4 | 7 |  | 1 | 8 | 0 | 5 | 6 |  | 9 | 2 | 4 | 8 | 2 |  | 1 | 5 | 4 | 2 | 2 |  |
| 96 | 2 | 6 | 6 | 4 | 5 |  | 3 | 9 | 9 | 4 | 8 |  | 6 | 3 | 8 | 5 | 5 |  | 7 | 3 | 9 | 9 | 3 |  | 0 | 7 | 6 | 0 | 2 |  | 0 | 3 | 5 | 7 | 5 |  | 6 | 3 | 3 | 5 | 0 |  |
| 97 | 6 | 8 | 3 | 4 | 1 |  | 9 | 5 | 5 | 4 | 0 |  | 1 | 4 | 9 | 1 | 6 |  | 1 | 1 | 6 | 8 | 5 |  | 7 | 0 | 5 | 0 | 4 |  | 2 | 7 | 3 | 1 | 6 |  | 8 | 7 | 2 | 2 | 9 |  |
| 98 | 0 | 3 | 9 | 9 | 5 |  | 5 | 5 | 2 | 3 | 2 |  | 6 | 2 | 2 | 8 | 0 |  | 9 | 0 | 3 | 9 | 3 |  | 1 | 2 | 3 | 8 | 8 |  | 2 | 9 | 9 | 9 | 7 |  | 4 | 8 | 8 | 0 | 4 |  |
| 99 | 5 | 4 | 6 | 7 | 7 |  | 8 | 3 | 2 | 0 | 6 |  | 2 | 7 | 3 | 5 | 1 |  | 1 | 9 | 3 | 4 | 7 |  | 2 | 7 | 3 | 3 | 9 |  | 9 | 2 | 3 | 5 | 5 |  | 8 | 1 | 8 | 6 | 1 |  |
| 100 | 4 | 4 | 7 | 2 | 5 |  | 4 | 6 | 9 | 5 | 0 |  | 1 | 4 | 9 | 6 | 7 |  | 0 | 0 | 4 | 7 | 7 |  | 3 | 7 | 5 | 5 | 2 |  | 4 | 8 | 4 | 7 | 2 |  | 9 | 2 | 8 | 2 | 8 |  |

**Teacher Notes**

New Illinois Learning Standards addressed by this activity:

Content Standards

|  |  |  |
| --- | --- | --- |
|   Supporting | 7.SP.A.2 | Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be*. |

Primary Math Practices

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

MP 4 Model with mathematics.

MP 5 Use appropriate tools strategically.

MP 6 Attend to Precision.

MP 7 Look for and make use of structure.

**Acknowledgements**

This activity is adapted from Beth Chance and Allan Rossman’s Sampling Words activity. Beth Chance and Allan Rossman have given permission for this activity and the corresponding applet to be shared with Illinois math teachers. The applet is available at <http://www.rossmanchance.com/applets/GettysburgSampleE/GettysburgSample.html>. Beth and Allan have also made a random number generator applet available at <http://www.rossmanchance.com/applets/RandomGen/GenRandom01.htm>.

**Guidance for Class Activity**

The video, ***7-SP.A.1 & 7-SP.A.2: Random Sampling to Draw Inferences About a Population, Part I***, available on the [www.ilstats.weebly.com](http://www.ilstats.weebly.com) website, includes a walk-through of the Gettysburg Address activity.

There are several key ideas that should be emphasized in this activity.

1. Students should know the terms population and sample and be able to identify the population and the sample. Students should understand what a simple random sample of size n is, know different ways of selecting a simple random sample, and be able to explain why samples should be selected randomly.
2. The average number of letters per word for the population of words in the Gettysburg Address is 4.295. In part I, students select a judgment sample. When selecting judgment samples, students tend to pick words that are typically longer than the population average.
3. To select a simple random sample of size 10, guide students in selecting ten random numbers either from a random digits table or from a random number generator.
4. The mean of a population is a characteristic (or parameter) of the population. The mean of a sample is a statistic. If we take several random samples from a population, we do not expect the mean of each of the random samples to be exactly the same. We expect them to vary. However, we anticipate that each of the sample means will be close to the population mean and therefore provide a reasonable estimate of the population mean.

**Random Digits Table**

To use a random digits table for this activity, students should first pick a row, and then select 3 digits at a time from this row. Numbers 001 to 268 correspond to the word ID. Skip over 000 and numbers greater than 268. Skip repeated numbers.

**Random Number Generator**

If students have access to computers, a random number generator can be used to generate 5 random numbers from 1 to 268. Beth Chance and Allan Rossman have shared this link to a random number generator that can be used for this activity. <http://www.rossmanchance.com/applets/RandomGen/GenRandom01.htm>

1. After creating a class dot plot of the means of random samples of size 10, use the Sampling Words applet found on the [www.rosssmanchance.com](http://www.rosssmanchance.com) website to demonstrate what happens to the shape and center (mean) of the sampling distribution as more and more samples of size 10 are selected from the population. The sampling distribution will be approximately normal with a mean that approaches the mean of the population. Imagine if the population were much larger, and it were not practical (or feasible) to find the actual population mean. A sampling distribution, created by taking many random samples from the population, will have a mean that provides a good estimate of the population mean. This allows us to make an inference about the population mean even though we cannot actually calculate the population mean.