**Margin of Error**

**Part I Do you Tweet?**

The Pew Internet Research Project, in an internet article titled *The Demographics of Social Media Users – 2012*, reported the results from a landline and cellphone survey of internet users. The table below is taken from the online article.

|  |  |
| --- | --- |
| Retrieved from <http://www.pewinternet.org/2013/02/14/the-demographics-of-social-media-users-2012/>. | Using the table at left, answer the following questions:1. What was the population of interest?
2. How many people were in the sample?

 **\_\_\_\_\_\_\_\_\_\_\_**1. What percent of all internet users use Twitter?

 **\_\_\_\_\_\_\_\_\_\_\_** 1. What **margin of error** is reported?

 **\_\_\_\_\_\_\_\_\_\_\_**1. What do you think this margin of error means?
 |

**Part II Paper Bag Population - Exploring the Margin of Error**

You have been given a paper bag that contains an entire population of colored beads. Without peeking inside the bag, what do you anticipate are the answers to each of the following questions. What colors are the beads in the bag? How many beads are in the bag? Are there any blue beads in the bag? If so, what proportion of the beads is blue? How do we answer these questions if we cannot see the entire population?

In real life, we are usually unable to make observations about an entire population. It is either not feasible or simply not practical to try to gather data from the whole population. To learn about the population, we take one or more samples from the population. If the sample(s) is representative of the population, we can make inferences about the population.

Imagine if we were interested in knowing what percentage of Illinois registered voters were going to vote for a particular candidate. Or suppose we wanted to know what percentage of all the students in your school favor starting and ending the school day an hour later. In this activity, we will use the paper bag population to simulate a real-life population. We cannot see the population, but we may take samples from the population. The proportion of a particular colored bead in the bag represents the proportion of voters voting for a particular candidate or the proportion of students favoring a change in the school day.

* What type of sample of your paper bag population should you take to ensure that the sample is representative?

Let’s conduct a statistical investigation.

 **Formulate a question**

What proportion of the beads in the bag is blue?

**Design and implement a plan to collect data**

We will take random samples of size 25 from the paper bag population. The sample proportion of blue beads will be our estimate of the proportion of blue beads in the paper bag population.

 Steps:

* Shake the bag to mix the beads. (Why are we doing this?)
* Without looking, reach in and draw one bead from the bag. Note the color and return the bead to the bag. This is called sampling with replacement. Shake the bag again and repeat until 25 draws have been made from the paper bag population.
* Count the number of blue beads in the sample and determine the proportion of blue beads in the sample.
* Plot this proportion on the class dot plot.
* Repeat until each student in the class has taken at least one turn.

|  |  |  |
| --- | --- | --- |
| **Bead Color** | **Tally** | **Frequency** |
| Blue |  |  |
| White |  |  |

What is your sample proportion of blue beads? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

This value will be your estimate of the proportion of blue beads in the paper bag population.

**Analyze the data by measures and graphs**

Copy the class dot plot here:

**Simulation of the Proportion of Blue Beads in the PaperBag Population**

**Sampling Distribution with Samples of Size 25**



 Proportion of Blue Beads in Sample

With your class, calculate the mean and standard deviation of the sampling distribution.

 Mean = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Standard Deviation = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Interpret the results in the context of the original question**

We know that samples vary. In the sampling distribution above, we can see that our sample proportions vary in a predictable way. In our work with sampling distributions we found that:

* The shape of the sampling distribution is: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* The center of the sampling distribution equals \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* The spread of the sampling distribution is less when \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* We expect \_\_\_\_\_\_\_% of our sample proportions to be within 2 standard deviations of the true population proportion.

|  |
| --- |
| **Margin of Error**One method of constructing an interval is to use a sample statistic ± a margin of error. The margin of error is 2 times the standard deviation of the sampling distribution.$$sample statistic\pm margin of error $$$$ sample mean\pm 2∙std Dev or sample proportion\pm 2∙std Dev $$$$where std Dev= standard deviation of the sampling distribution $$We use the reasoning that, if the sample statistic is likely to be within 2 standard deviations of the center of the sampling distribution, then the center of the distribution is also likely to be within 2 standard deviations of the sample statistic. Remember that the population parameter is approximately equal to the mean of the sampling distribution, so the population parameter is likely to be within two standard deviations of our sample statistic. In other words, we conclude that the population parameter is likely to be in this interval. (Or at least we expect 95% of the intervals created by this method to include the population parameter.)  |

* What is our margin of error?

Margin of error = $ \pm 2∙std Dev $

 = $\pm 2∙$ \_\_\_\_\_

 = \_\_\_\_\_\_\_\_\_

* Report a margin of error using your sample proportion as the estimate for the population proportion. Your estimate should be given as:

$$sample proportion\pm margin of error $$

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* The true population proportion of blue beads in the paper bag population lies between what two values?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Part III Back to Tweeting**

In the Tweeting survey, a percent rather than a proportion is used.

1. What is the estimated value of the percent of internet users that use Twitter? \_\_\_\_\_\_\_
2. What is the reported margin of error? \_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Write a sentence interpreting the margin of error for this survey.

**Part IV Practice Problems**

Problem 1 Hours of Homework

How many hours do American high school students spend per week on homework? To answer this question, a class of 28 students each took random samples of 25 United States high school students from the data base at Census at School (<http://www.amstat.org/censusatschool/index.cfm>). The dot plot below shows the distribution of the averages for their samples.



1. Based on the standard deviation of the sampling distribution above, what is the estimated margin of error?
2. One of the students in the class had a sample with a mean of 11.0 hours. Based on this sample mean, we anticipate the actual average number of hours spent on homework by United States high schools students who completed the Cenusus at School survey to fall within what interval?
3. If each of the 28 students constructed his own interval by calculating sample mean$ \pm 2∙std Dev$, would we expect the actual population mean to fall within each of the 28 intervals? Explain.
4. If the students had taken samples of size 50 instead of samples of size 25, how would this change the interval that you found in (b) above?

Problem 2 President Obama’s Job Approval Rating





From Gallup: <http://www.gallup.com/poll/113980/Gallup-Daily-Obama-Job-Approval.aspx>

1. What was the population of interest in this statistical study?
2. How was the sample selected?
3. How many people were in the sample?
4. Assuming that the sample of 1500 adults who were polled is representative of the American population, we infer that the actual percentage of Americans who approve of the job President Obama is doing as president is likely to be between what two values?
5. Is it likely that the actual percentage of Americans that approve of the job that President Obama is doing as president is 50% or greater? Explain.

1. If Gallop took a smaller sample, how would this change your answer to part (d)? Explain.

Problem 3 Job Satisfaction

Three large international companies ask all their employees to take an online job satisfaction survey offered by Jobs Plus, a placement firm. An employee’s job satisfaction score from the survey is represented by a value from 0 to 100. Jobs Plus provides a written summary of the survey results back to each company. The companies can then choose what information they wish to share as part of their recruiting process. Below is information shared by each of the three companies.

|  |
| --- |
| **Company A**The graph below shows the distribution of the mean job satisfaction score for 100 random samples of 25 employees across all branches of our company.  |
| **Company B**Mean Job Satisfaction Score: 70.2 ± 3.8(Based on a single random sample of size 25.) |
| **Company C**Mean Job Satisfaction Score: 62.3 ± 1.2(Based on a single random sample of size 25.) |

1. What is the best estimate of the population mean job satisfaction score for company A? Explain.
2. Suppose Company A had decided to report a margin of error rather than share a sampling distribution. What is the margin of error for the mean job satisfaction score of Company A employees, if the mean from a single random sample of 25 Company A employees is 67.4? Write a sentence interpreting this margin of error in context.
3. How would the shape, center and spread of Company A’s sampling distribution change if the sample size was increased to 40 employees?
4. How would company A’s margin of error in (b) change if the sample size was increased to 40 employees?
5. Based on the information shared by Company A and B, is there evidence to conclude that the actual mean job satisfaction score for company B is higher than the mean job satisfaction score for company A? Explain.
6. Is there evidence to conclude that Company A employees, on average, have a higher job satisfaction than Company C employees?

**Teacher Notes**

New Illinois Learning Standards addressed by this activity:

Content Standards

|  |  |  |  |
| --- | --- | --- | --- |
| Major | S | IC.4 | Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.\* |

Primary Math Practices

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

MP 4 Model with mathematics.

MP 5 Use appropriate tools strategically.

MP 6 Attend to Precision

MP 7 Look for and make use of structure.

Acknowledgements

Duggan, M. & Brenner, J. (2013, February 14). *The Demographics of Social Media Users – 2012*. Retrieved from <http://www.pewinternet.org/2013/02/14/the-demographics-of-social-media-users-2012/>.

Census at School <http://www.amstat.org/censusatschool/>

Thank you to Roxy Peck for proof reading the two sample problems and providing guidance on the simulation to determine and explanation of the margin of error.

Guidance for Activity

The paper bag population may consist of any uniformly-shaped, small item. Beads, small Legos, colored chips, or dried beans all work well. Determine ahead of time a value for the population proportion and make each of the paper bags have the same proportion. Values between 0.3 and 0.7 for the population proportion work best. A suggested population for the individual student population bags is to use 100 beads with 35% white and 65% blue. Technically, when sampling with replacement, each student paper bag population could be limited to 10 or 20 beads. Using replacement simulates having a large (unlimited) population in which the proportion of interest remains constant. The main advantage to using 100 beads is that it makes it more difficult for a student who is inclined to peek to calculate the population proportion of blue beads.

Another version of this activity is available at <http://www.ilclassroomsinaction.org/> in which a single population bag with a larger population of beads is used for the entire class. A larger population helps the students to realize that counting the entire population would not be a good method. In this second version, each student draws a single random sample from the class population bag, calculates the sample proportion and then shares the sample proportion with the class. When a single class population bag is used, time is needed for each student to take a random sample. Consider having a separate activity or review task that students can work on while samples are being taken.

You may want to assign students to read the internet article, *The Demographics of Social Media Users – 2012* as part of their work for S.IC.3 and 6. (See <http://www.pewinternet.org/files/old-media//Files/Reports/2013/PIP_SocialMediaUsers.pdf>)

An online applet can be used to illustrate margin of error. Explain that since we expect 95% of the sample proportion to be within 2 standard deviations of the population proportion (the center of the sampling distribution), then we expect the population proportion to be within 2 standard deviations of most of the sample proportions. We use the sample proportion +/- a margin of error to estimate the population proportion. When the sample proportion is used as a prediction of the population proportion, the symbol $\hat{p}$ (p-hat) is used. Students can either think of the margin of error as

 sample statistic ± margin of error

 or as

 $\hat{p} $± margin of error $\overbar{x}$± margin of error

where

 margin of error = 2$∙standard deviation of the sampling distribution ( std dev of estimate)$

Students are not expected to know the statistical variables $\hat{p}$ or $\overbar{x}$ as part of their work on the Common Core State Standards. This notation will not be a part of the PARCC assessments. They are shown here only as information for you as an instructor.

Below are example sampling distributions for 28 samples of size of 25 for population bags with population proportions ranging from 0.30 to 0.70.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

**Answer Key**

Part I Do you Tweet?

1. *Internet users*
2. *1802*
3. *16%*
4. *± 2.6%*
5. *(Answers will vary.) Students will return to this question in Part III*

Part II paper Bag Population

* *If we want the sample to be representative of the population, a random sample should be taken.*

Interpret the results

* *The shape of the sampling distribution is approximately normal.*
* *The center of the sampling distribution equals the population proportion.*
* *The spread of the sampling distribution is less when the sample size is increased.*
* *95%*
* What is the margin of error?

(Answers will vary.)

Sample response from a population with a proportion of 0.65 red.

*Std Dev ≈ 0.095*

*Margin of error = ± 2 ∙ 0.095*

 *= ± 0.190*

* 0.56 *± 0.190*
* *The true populati0n proportion of blue beads in the paper bag population is likely to be on the interval 0.37 and 0.75 . (0.56 - 0.19 = 0.37 and 0.56 + 0.19 = 0.75)*

*Note: If a larger sample size was used, the width of this interval would decrease.*

*The word “likely” is used because we anticipate 95% of our random samples to fall within 2 standard deviations of our the population proportion, so the population proportion should fall within 2 standard deviations of 95% of our random samples.*

Part III Back to Tweeting

1. 16%
2. *± 2.6%*
3. *The true proportion of internet users that tweet is likely to be on the interval 13.4% to 18.6%.*

Part IV Practice Problems

Problem 1 Hours of Homework

1. *± 2 ∙ 1.67088*

*= ± 3.34176*

*≈ ± 3.34*

1. *11.0 ± 3.34*

*The actual average number of hours spent on homework by U.S. high school students who completed the Census at School survey is likely to lie on the interval 7.66 to*

1. *No. We expect approximately 95% of the sample means to be within 2 standard deviations of the population mean, so the population mean will be within 2 standard deviations of 95% of the means of our random samples.*
2. *A larger sample size will produce a smaller standard deviation, a smaller margin of error, and, therefore, a narrower interval.*

Problem 2 President Obama’s Job Approval Rating

1. *The population of interest is probably U.S. adults. However, the actual population is U.S. adults who have a phone.*
2. *No information is given about how the sample was selected. It may or may not have been selected randomly.*
3. *1500 adults*
4. *42 % ± 3 %*

*The actual percentage of U.S. adults (with phones) who approve of the job President Obama is doing is likely to be on the interval 39% to 45%.*

*Based on the margin of error in d, if the sample was selected randomly, it is unlikely that the percentage of Americans that approve of the job that President Obama is doing is 50% or greater. 50% is not on the interval 39% to 45%.*

1. *If Gallop took a smaller sample, the margin of error would increase and the interval would be wider.*

Problem 3 Job Satisfaction

1. *The center of the sampling distribution provides a very good estimate of the population mean. The mean job satisfaction score for the population of all Company A employees is approximately 67.892.*
2. *Mean ± 2∙ std Dev*

 *67.4 ± 2∙ 1.76*

 *67.4 ± 3.52*

*63.88 to 70.92*

*The mean job satisfaction score for the population of Company A employees is likely to be on the interval 63.88 to 70.92.*

*(Note: The mean of the sampling distribution provides us with a very good estimate of the population mean. However, if we did not have the actual sampling distribution for Company A, then a random sample of Company A employees and a margin of error provides us an interval on which we anticipate to find the population mean.)*

1. *The shape will still be approximately normal and the center will still be approximately 67.89. However, the spread of the distribution will decrease. (The standard deviation will be smaller.)*
2. *The standard deviation will decrease, so the margin of error will also decrease.*
3. *Company B: 70.2 ± 3.8*

 *66.4 to 74.0*

*The mean job satisfaction score for company B employees is likely to lie on the interval 66.4 to 74.0. This interval includes the center of the Company A sampling distribution. It is possible that the actual population mean job satisfaction score for Company B can be anywhere on the interval 66.4 to 74.0, so it is possible that Company B’s population mean job satisfaction score could be higher or lower (or equal) to Company A’s population mean job satisfaction score.*

1. *Company C: 62.3 ± 1.2*

 *61.1 to 63.5*

*The mean job satisfaction score for Company C is likely to lie on the interval 61.1 to 63.5. This interval is below almost all (or possibly all) the random sample means shown in the sampling distribution for Company A. It is very likely that Company A’s population mean job satisfaction score is higher than Company C’s mean population job satisfaction score.*